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New prioritization of QFDs' engineering characteristics through the Law of Comparative Judgment

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The *House of Quality* (HoQ) is the key module of *Quality Function Deployment* (QFD), which aims to support the design of new products or services by translating customer requirements (CRs) into engineering characteristics (ECs). The *independent scoring method* is the most widely used technique for the prioritisation of ECs; despite this, it has two conceptual shortcomings: (i) arbitrary numerical conversion of (ordinal) relationships between CRs and ECs (i.e., *null*, *weak*, *medium*, and *high*), and (ii) "promotion" of these relationships from an *ordinal* to a *cardinal* scale. This paper introduces a novel procedure for prioritizing ECs, inspired by the Thurstone's *Law of Comparative Judgment* (LCJ), which overcomes the conceptual shortcomings of the ISM while maintaining its flexibility and ease of implementation. A realistic application example illustrates the potential of the proposed procedure.

Keywords: house of quality; engineering-characteristic prioritization; law of comparative judgment; scale promotion.

1. Introduction

Quality Function Deployment (QFD) is a powerful methodology for designing new products and services according to *customer requirements* (CRs) (Franceschini, 2001). The *House of Quality* (HoQ) is the initial QFD module, translating CRs into *engineering characteristics* (ECs) [1]. The core of the HoQ is the *relationship matrix*, which establishes the relationships between ECs and CRs, through symbols indicating the intensity of the relationship: ∅ → *None*, Δ → *Low*, ○ → *Medium*, ● → *High*).

Figure 1 exemplifies a relationship matrix for the design of a climbing safety harness. The eight CRs shown in the rows on the left have been associated with weights, reported on the right (i.e., w_{CR_i} values, being $\sum_{\forall i} w_{\mathrm{CR}_i} = 1$), which describe their relevance to the final customer [2]. The columns of the relationship matrix refer to the ECs (i.e., EC_1, EC_2, ...) associated with the CRs. The relationship between ECs and CRs is expressed through the aforementioned symbols.

Rel. intensity	Symbol	r_{ij}
None	∅	0
Low	△	1
Medium	○	3
High	●	9

		Meets safety standards EC1	Harness weight EC2	Webbing strength EC3	No. of colours EC4	No. of sizes EC5	Padding thickness EC6	No. of buckles EC7	No. of gear loops EC8	w_{CR_i}
Easy to put on	CR1					○		●		7%
Comfortable when hanging	CR2					○	●	○		20%
Fits over different clothes	CR3					○	○	●		3%
Accessible gear loops	CR4								●	12%
Does not restrict movement	CR5		○			○	●	○		27%
Lightweight	CR6		●	○			○	△	△	10%
Safe	CR7	●	○	●						14%
Attractive	CR8		△		●		△	△		7%
	w_{EC_j}	1.26	2.20	1.56	0.63	1.71	4.69	2.48	1.18	

Fig. 1. Relationship matrix and EC prioritization using the ISM.

One of the objectives of the HoQ is to prioritize the ECs in order to identify the CRs that have the greatest impact on the final customer, so that more attention can be paid to them in the design process. In general, more attention should be given to the ECs that are related to multiple CRs, with relatively high weights, and high intensity relationships. The prioritization technique traditionally used is the *independent scoring method* (ISM), i.e., a weighted sum of coefficients obtained by numerical conversion of the symbols in the relationship matrix, using the w_{CR_i} values as weights. The resulting weight of the *j*th EC is determined by:

$$w_{\text{EC}_j} = \sum_{\forall i}\left(r_{ij} \cdot w_{\text{CR}_i}\right), \quad (1)$$

where r_{ij} is the numerical coefficient corresponding to the relationship between EC_j and CR_i, according to the numerical (exponential) conversion ∅ → 0, Δ → 1, ○ → 3, ● → 9 (cf. top left of Fig. 1).

Although ISM is simple and practical, it has some shortcomings. First, the conversion of the relationship-matrix symbols (i.e., *ordinal* quantities) into numerical coefficients is arbitrary. Secondly, the aggregation of these numerical coefficients through a weighted sum improperly "promotes" them from ordinal to *cardinal* quantities [3]. These shortcomings make the solution provided by ISM conceptually imperfect [4, 5].

The objective of this contribution is to develop a new technique for prioritizing ECs that is alternative to ISM and overcomes its weaknesses. The new procedure will be based on Thurstone's *Law of Comparative Judgment* (LCJ), which is a well-established, conceptually rigorous, and effective statistical technique [6].

2. Methodology

This section briefly revisits the LCJ, contextualizing it within the framework of the QFD's HoQ. In general, LCJ aims to prioritize a set of objects (i.e., ECs) based on an attribute (i.e., their impact on the final customer, from the perspective of the individual CRs). Each CR — which corresponds to a specific row of the relationship matrix — can be translated into paired-comparison relationships among potential pairs of ECs. This process takes place in two stages. In the first stage, a ranking of ECs is identified for each CR, based on their impact on the CR itself. Let us illustrate this with an example related to CR_1 (cf. the first row of the relationship matrix in Fig. 1). Based on the symbols denoting intensities of relationship with the ECs, a ranking consisting of four preference blocks (in round brackets) with mutual *strict preference* relationships ("≻") can be constructed:

$$(EC_\bullet \sim EC_7) \succ (EC_\circ \sim EC_5) \succ EC_\triangle \succ (EC_\emptyset \sim EC_1 \sim EC_2 \sim EC_3 \sim EC_4 \sim EC_6 \sim EC_8). \quad (2)$$

The block $(EC_\bullet \sim EC_7)$, including ECs with high-intensity relationship is followed by the one $(EC_\circ \sim EC_5)$ including ECs with medium-intensity relationship, then the one $(EC_\triangle)$ including ECs with weak-intensity relationship, and finally the one $(EC_\emptyset \sim EC_1 \sim EC_2 \sim EC_3 \sim EC_4 \sim EC_6 \sim EC_8)$ including ECs with no relationship. This ranking also includes four special (or *dummy*) objects to account for the relationship intensity: $EC_\emptyset$ → *Absence of impact*, $EC_\triangle$ → *Low impact*, $EC_\circ$ → *Medium impact*, and $EC_\bullet$ → *Maximum-possible impact*.

Table 1. Paired-comparison relationships between ECs, from the perspective of their impact on CR_1.

	EC_1	EC_2	EC_3	EC_4	EC_5	EC_6	EC_7	EC_8	$EC_\emptyset$	$EC_\triangle$	$EC_\circ$	$EC_\bullet$
EC_1	-											
EC_2	$\sim$	-										
EC_3	$\sim$	$\sim$	-									
EC_4	$\sim$	$\sim$	$\sim$	-								
EC_5	$\prec$	$\prec$	$\prec$	$\prec$	-							
EC_6	$\sim$	$\sim$	$\sim$	$\sim$	$\succ$	-						
EC_7	$\prec$	$\prec$	$\prec$	$\prec$	$\prec$	$\prec$	-					
EC_8	$\sim$	$\sim$	$\sim$	$\sim$	$\succ$	$\sim$	$\succ$	-				
$EC_\emptyset$	$\sim$	$\sim$	$\sim$	$\sim$	$\succ$	$\sim$	$\succ$	$\sim$	-			
$EC_\triangle$	$\prec$	$\prec$	$\prec$	$\prec$	$\succ$	$\prec$	$\succ$	$\prec$	$\prec$	-		
$EC_\circ$	$\prec$	$\prec$	$\prec$	$\prec$	$\sim$	$\prec$	$\succ$	$\prec$	$\prec$	$\prec$	-	
$EC_\bullet$	$\prec$	$\prec$	$\prec$	$\prec$	$\prec$	$\prec$	$\sim$	$\prec$	$\prec$	$\prec$	$\prec$	-

In the second step, each ranking is translated into paired-comparison relationships, as exemplified in Table 1, with reference to CR_1 [3]. These paired comparisons are based on the intensity of the relationship with the CR of interest, utilizing relationships of *strict preference* (e.g., "$EC_j \succ EC_k$" and "$EC_j \prec EC_k$") as well as *indifference* (e.g., "$EC_j \sim EC_k$").

The same construction can be extended to the other CRs (i.e., CR_2 to CR_8), resulting in eight total sets of paired-comparison relationships (omitted for the sake of brevity). For each specific *jk*th paired comparison of ECs (i.e., EC_j versus EC_k), the eight available relationships can be aggregated on the basis of the weights of the CRs associated with them. With reference to each *i*th CR and each *jk*th paired comparison, the following three *binary parameters* are defined:

$$
\begin{aligned}
&c_{i,jk}^{(\succ)} \text{ being equal to 1 if } EC_j \succ EC_k \text{ (otherwise 0)},\\
&c_{i,jk}^{(\sim)} \text{ being equal to 1 if } EC_j \sim EC_k \text{ (otherwise 0)},\\
&c_{i,jk}^{(\prec)} \text{ being equal to 1 if } EC_j \prec EC_k \text{ (otherwise 0)}.
\end{aligned}
\tag{3}
$$

It can be seen that these parameters are mutually exclusive, and the complementarity relationship holds: $c_{i,jk}^{(\succ)} + c_{i,jk}^{(\sim)} + c_{i,jk}^{(\prec)} = 1$. A general parameter expressing the degree of preference of EC_j over EC_k, from the perspective of the *i*th CR, can be defined as:

$$c_{i,jk} = 1 \cdot c_{i,jk}^{(\succ)} + ½ \cdot c_{i,jk}^{(\sim)} + 0 \cdot c_{i,jk}^{(\prec)}, \in \{0, ½, 1\}. \tag{4}$$

Finally, the $c_{i,jk}$ parameters are utilised in the following weighted sum, which allows to determine the p_{jk} values:

$$p_{jk} = \sum_{\forall i}\left(c_{i,jk} \cdot w_{CR_i}\right), \in [0, 1]. \tag{5}$$

These p_{jk} values represent the weighted portion of CRs on which EC_j has a greater impact than EC_k.[a] By construction, the complementary relationship $p_{jk} = 1 - p_{jk}$ applies. All p_{jk} values, calculated for all possible *jk*th paired comparisons, are aggregated into a matrix of proportions (***P***), where symmetrical elements with respect to the main diagonal complement each other.

Let us briefly digress to better understand the practical meaning of the p_{jk} values. The LCJ is based on several postulates, including the representation of the impact of ECs on the final customer using a latent one-dimensional scale.

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[a]In the traditional LCJ procedure, p_{jk} proportions are determined based on the judgments on the objects in question by a panel of experts, without the weighting introduced in this adaptation.

Specifically, the impact of a specific EC (such as EC_j) can be treated as a normally distributed variable, with an unknown mean and variance, reflecting its variability across different CR perspectives: $\mathrm{EC}_j \sim \mathbb{N}\left(\mu_{\mathrm{EC}_j}, \sigma^2_{\mathrm{EC}_j}\right)$. Additionally, the LCJ assumes equal variances for ECs ($\sigma^2_{\mathrm{EC}_1} = \sigma^2_{\mathrm{EC}_2} = \ldots = \sigma^2$), and equal intercorrelations between pairs of ECs (in the form of Pearson coefficients $\rho_{\mathrm{EC}_j,\mathrm{EC}_k} = \rho, \forall j, k$). The difference between the impact of EC_j and EC_k can thus, by extension, be represented as a normally distributed variable with the following parameters: $\left(\mathrm{EC}_j - \mathrm{EC}_k\right) \sim \mathbb{N}\left[\left(\mu_{\mathrm{EC}_j} - \mu_{\mathrm{EC}_k}\right), 2 \cdot \sigma^2 \cdot (1 - \rho)\right]$ (Thurstone, 1928). Now, focusing on the probability:

$$P\left[\left(\mathrm{EC}_j - \mathrm{EC}_k\right) > 0\right] = P\left(\mathrm{EC}_j > \mathrm{EC}_k\right), \tag{6}$$

it represents the portion of the CRs where EC_j has more impact than EC_k and precisely corresponds to the quantity p_{jk} that we determined earlier. Therefore, this implies that p_{jk} can be expressed as a function of the unknown values μ_{EC_j} and μ_{EC_k} (demonstration omitted).

Next, the p_{jk} values are transformed into z_{jk} values using the relationship:

$$z_{jk} = \Phi^{-1}(1 - p_{jk}), \tag{7}$$

$\Phi^{-1}(\cdot)$ being the inverse of the cumulative distribution function of the standard normal distribution, and z_{jk} being a unit normal deviate. In general, p_{jk} values of 1.00 and 0.00 would correspond to z_{jk} values of $\pm\infty$; a simplified approach to address this issue is to associate p_{jk} values ≤ 0.00135 with $z_{jk} = \Phi^{-1}(1 - 0.00135) = 3$ and p_{jk} values ≥ 0.99865 with $z_{jk} = \Phi^{-1}(1 - 0.99865) = -3$. More sophisticated solutions to deal with this issue have been proposed [3].

Next, the z_{jk} values related to the possible paired comparisons are reported into a matrix $\boldsymbol{Z}$. The relationship $z_{kj} = -z_{jk}$ holds, being unit normal deviates related to complementary cumulative probabilities. It can be demonstrated that the unknown mean values of the impact of individual ECs (μ_{EC_j}) can be determined — up to a positive scale factor and an additive constant — by summing column by column the values contained in the $\boldsymbol{Z}$-matrix. In other words, the LCJ allows for to be positioned of ECs on an *interval* scale (referred to as x) with an arbitrary zero point and unit.

Drawing inspiration from the methodology developed in [3], the interval scaling resulting from the LCJ can be “anchored”, using two of the previously introduced dummy objects: $EC_\emptyset$, which represents the absence of a relationship, and $EC_\bullet$, which represents the maximum-possible degree of intensity of a relationship with the CR of interest. The (interval) scale x is then transformed into a new one (y), defined within the conventional range [0, 10], through the following linear transformation:

$$\frac{y_k - 0}{10 - 0} = \frac{x_k - x_\emptyset}{x_\bullet - x_\emptyset} \rightarrow y_k = 10 \cdot \frac{x_k - x_\emptyset}{x_\bullet - x_\emptyset}, \tag{8}$$

where:

$x_\emptyset$ and $x_\bullet$ are the scale values of $EC_\emptyset$ and $EC_\bullet$, resulting from LCJ;

x_k is the scale value of a generic kth EC, resulting from LCJ;

y_k is the transformed scale value within the conventional range [0, 10].

The introduction of $EC_\emptyset$ and $EC_\bullet$ allows to anchor the scale x into a new scale (y) with a conventional unit and a zero point (which corresponds to the absence of the attribute); it is therefore admissible to consider y as a *ratio* scale and use the y-values of the ECs as w_{EC_k} values (cf. Sec. 1).

Figure 2 summarises the results of the application of the proposed procedure to the HoQ in Fig. 1.

3. Concluding remarks

This research presented an innovative procedure for prioritizing ECs within the HoQ framework, building upon Thurstone's LCJ. This procedure offers several notable advantages, including its ability to adapt to the traditional relationship matrix without the need for specialized response modes. It prioritizes ECs using a *ratio* scaling approach, anchored by the inclusion of dummy ECs. In addition, it is easy to implement, automatable, flexible, and adaptable to other response modes than the traditional one. The effectiveness and robustness of the new EC prioritization is ensured by the LCJ itself, which is a well-established technique that has been used and tested in multiple contexts [3].

In the test case, the new procedure closely aligns with ISM, demonstrating a high correlation (a comparison of the w_{EC_k} values resulting from the two methods yields $R^2 \approx 0.8602$) and producing EC rankings that are nearly identical, with only a minor rank reversal between EC_1 and EC_8:

$$\begin{aligned} &\text{ISM: } EC_6 > EC_7 > EC_2 > EC_5 > EC_3 > EC_1 > EC_8 > EC_4. \\ &\text{New procedure: } EC_6 > EC_7 > EC_2 > EC_5 > EC_3 > EC_8 > EC_1 > EC_4. \end{aligned} \tag{9}$$

While preliminary tests showed some agreement between the proposed procedure and the ISM, we acknowledge the possibility of divergent results in specific scenarios, that will be subject to further investigation. Looking ahead, the proposed procedure represents a valuable addition to the QFD toolkit, expanding the analytical possibilities. Despite the inherent limitations of LCJ [3], it is planned to explore its adaptation for situations with uncertain and/or incomplete formulation of relationships. Additionally, it is planned to conduct comprehensive comparative analyses with alternative ISM methodologies from existing QFD literature [7].

(a) Matrix ***P***

	EC_1	EC_2	EC_3	EC_4	EC_5	EC_6	EC_7	EC_8	$EC_{\emptyset}$	$EC_{\triangle}$	$EC_{\circ}$	$EC_{\bullet}$
EC_1	0.500	0.350	0.450	0.535	0.285	0.235	0.200	0.460	0.570	0.140	0.140	0.070
EC_2	0.650	0.500	0.650	0.720	0.505	0.370	0.470	0.730	0.790	0.545	0.305	0.050
EC_3	0.550	0.350	0.500	0.585	0.335	0.285	0.300	0.560	0.620	0.240	0.190	0.070
EC_4	0.465	0.280	0.415	0.500	0.250	0.235	0.200	0.425	0.535	0.070	0.070	0.035
EC_5	0.715	0.495	0.665	0.750	0.500	0.215	0.365	0.675	0.785	0.570	0.285	0.000*
EC_6	0.765	0.630	0.715	0.765	0.785	0.500	0.735	0.775	0.835	0.635	0.535	0.235
EC_7	0.800	0.530	0.700	0.800	0.635	0.265	0.500	0.760	0.870	0.655	0.335	0.050
EC_8	0.540	0.270	0.440	0.575	0.325	0.225	0.240	0.500	0.610	0.170	0.120	0.060
$EC_{\emptyset}$	0.430	0.210	0.380	0.465	0.215	0.165	0.130	0.390	0.500	0.000*	0.000*	0.000*
$EC_{\triangle}$	0.860	0.455	0.760	0.930	0.430	0.365	0.345	0.830	1.000*	0.500	0.000*	0.000*
$EC_{\circ}$	0.860	0.695	0.810	0.930	0.715	0.465	0.665	0.880	1.000*	1.000*	0.500	0.000*
$EC_{\bullet}$	0.930	0.950	0.930	0.965	1.000*	0.765	0.950	0.940	1.000*	1.000*	1.000*	0.500

(b) Matrix ***Z***

	EC_1	EC_2	EC_3	EC_4	EC_5	EC_6	EC_7	EC_8	$EC_{\emptyset}$	$EC_{\triangle}$	$EC_{\circ}$	$EC_{\bullet}$
EC_1	0.000	0.385	0.126	-0.088	0.568	0.722	0.842	0.100	-0.176	1.080	1.080	1.476
EC_2	-0.385	0.000	-0.385	-0.583	-0.013	0.332	0.075	-0.613	-0.806	-0.113	0.510	1.645
EC_3	-0.126	0.385	0.000	-0.215	0.426	0.568	0.524	-0.151	-0.305	0.706	0.878	1.476
EC_4	0.088	0.583	0.215	0.000	0.674	0.722	0.842	0.189	-0.088	1.476	1.476	1.812
EC_5	-0.568	0.013	-0.426	-0.674	0.000	0.789	0.345	-0.454	-0.789	-0.176	0.568	3.000
EC_6	-0.722	-0.332	-0.568	-0.722	-0.789	0.000	-0.628	-0.755	-0.974	-0.345	-0.088	0.722
EC_7	-0.842	-0.075	-0.524	-0.842	-0.345	0.628	0.000	-0.706	-1.126	-0.399	0.426	1.645
EC_8	-0.100	0.613	0.151	-0.189	0.454	0.755	0.706	0.000	-0.279	0.954	1.175	1.555
$EC_{\emptyset}$	0.176	0.806	0.305	0.088	0.789	0.974	1.126	0.279	0.000	3.000	3.000	3.000
$EC_{\triangle}$	-1.080	0.113	-0.706	-1.476	0.176	0.345	0.399	-0.954	-3.000	0.000	3.000	3.000
$EC_{\circ}$	-1.080	-0.510	-0.878	-1.476	-0.568	0.088	-0.426	-1.175	-3.000	-3.000	0.000	3.000
$EC_{\bullet}$	-1.476	-1.645	-1.476	-1.812	-3.000	-0.722	-1.645	-1.555	-3.000	-3.000	-3.000	0.000

(c) Scaling

	EC_1	EC_2	EC_3	EC_4	EC_5	EC_6	EC_7	EC_8	$EC_{\emptyset}$	$EC_{\triangle}$	$EC_{\circ}$	$EC_{\bullet}$
$x_k = \Sigma_k$	-6.116	0.336	-4.167	-7.989	-1.627	5.202	2.161	-5.794	-13.545	0.183	9.025	22.330
$w_{EC_k} = y_k$	2.1	3.9	2.6	1.5	3.3	5.2	4.4	2.2	0.0	3.8	6.3	10.0

Fig. 2. (a) ***P*** matrix, (b) ***Z*** matrix, and (c) scaling resulting from the application of the proposed procedure to the test case. $x_k = \Sigma_k$ represents the summation of the values reported in the kth column of the ***Z*** matrix and corresponds to the *interval* scale value resulting from LCJ; the y_k values relate to the *ratio* scaling downstream of anchoring in Eq. (8).

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